1: The Taylor series

Let consider the mathematical function:

$$y = sin(x)$$

(a) Solve:

Derive the Mac-Laurin approximations (Taylor series in the point 0) of order 0, 1, 2, and 3.
hint 1: $sin'(x) = cos(x)$
hint 2: $cos'(x) = -sin(x)$

(b) R commands:

- Defines an R function for each of the 4 approximations
- Plot the function and its four Taylor approximations

2: The Normal likelihood function

Let consider the log-likelihood function for the normal r.v.:

$$l(\theta) = ln(\theta) = -\frac{n}{2}ln(\sigma^2) - \frac{n}{2}ln(2\pi) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2$$

(a) Solve:

1. Case with $\theta = \mu$; σ^2 known
   - Formulate the expression of the score function (first derivative of the log–likelihood)
   - Derive the maximum likelihood estimate
   - Derive the Fisher observed information
2. Case with $\theta = \sigma^2$; μ known
   - Formulate the expression of the score function (first derivative of the log–likelihood)
   - Derive the maximum likelihood estimate
   - Derive the Fisher observed information

(b) R commands:

Let X be a normal r.v. with mean μ and $\sigma^2 = 1$ and consider the following observed sample:
-0.34 1.40 2.38 4.16 4.65

- **Solve:** Formulate the expression of the likelihood function
- Write an R–function for the likelihood function
- Plot the likelihood function for the given sample

- **Solve:** Formulate the expression of the log–likelihood function
- Write an R–function for the log–likelihood function
- Plot the log–likelihood function for the given sample